VISCOUS ATTENUATION OF SOLITARY INTERNAL WAVES IN A TWO-LAYER FLUID

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In the present article we report an experimental study of the attenuation of solitary internal waves at an interface between two fluids of different densities under the influence of viscosity. This problem has been investigated theoretically in [1], where the results of [2] are generalized to the case of a covered two-layer fluid. Motion with a free surface has been investigated in [3]. A problem similar to the one in [3] has been studied [4] by the numerical solution of a model integrodifferential equation. The attenuation of solitary internal waves in a two-layer fluid has been investigated [1, 5]. It is important to note that the experiments in [1] were performed at an interface between immiscible fluids with a free surface, whereas the theoretical analysis is carried out for the covered-flow case. The experiments reported in [5], from which the results were used for comparison with the theoretical analysis in [3], were carried out with miscible fluids, whereas interfacial friction, which was ignored in [5], is taken into account in [3]. The experiments in the present study were performed with immiscible fluids, both with a free surface and covered. The results are compared with theoretical models.

The experiments were set up on three different apparatus configurations, which are shown schematically in Fig. 1. One arrangement comprised an open channel (Fig. 1a) of length 220 cm, width 17.5 cm, and height 15 cm; the other two were end-damped tubes with cross sections of 6 × 6 and 20 × 6 cm and a length of 390 cm (Fig. 1b); h is the depth of the lower fluid, H is the total depth of both layers, and a is the wave amplitude. The lower fluid was a dilute solution of sodium chloride in distilled water with a density $\rho = 1$ g/cm^3 and viscosity $\nu = 0.0108$ cm^2/sec. The upper layer was either kerosene with $\rho_1 = 0.8$ g/cm^3 and $\nu_1 = 0.0162$ cm^2/sec or a mixture of kerosene and Khladon-113 (a coolant) with $\rho_1 = 0.9$ g/cm^3 and $\nu_1 = 0.0141$ cm^2/sec (pure Khladon-113 has a density of 1.58 g/cm^3 and a viscosity of 0.0044 cm^2/sec).

Two techniques were used for wave generation. In one case the plate 1 (Fig. 1a) was placed at the end wall of the channel, where it could be moved vertically. When it was moved down or up, a solitary wave propagated away from it in the form of a hump or a trough (the example of generation of a hump is shown in Fig. 1a). The wave suppressor 3 was set up in the free-surface experiments, where it was needed in order to dampen the oscillations excited by the motion of the wave generator (particularly for the generation of a wave in the form of a trough). The second technique entailed the following: A 60-cm-long section of the channel was isolated by the partition 1 (Fig. 1b), at which a level difference Δh was created. When the partition was lifted briefly, a solitary wave in the form of a trough propagated away from one side of it, and a solitary hump wave propagated away from the other side.

The waves were recorded by the electrical conductivity sensors 2 [6], which were spaced along the channel at equal intervals Δx. The first sensor was set up at a distance x_0 from the wave generator, which is far enough to permit the solitary wave to form completely and to ensure the decay of unwanted short-wavelength interfacial oscillations accompanying wave generation. The error of measurement of waves of amplitude 0.5 mm was less than or equal to 5% and decreased as the amplitude was increased.

The discussion of the experimental results must be prefaced by a few remarks pertaining to the tested theoretical models. The same dependence of the attenuation of solitary internal waves on the distance traversed is given in [1, 3], viz.:

$$a(x) = a_0(x_0)\left[1 + Ka_0^{1/4}(x - x_0)\right]^{-4}, \tag{1}$$

but different relations are given for the coefficient K.

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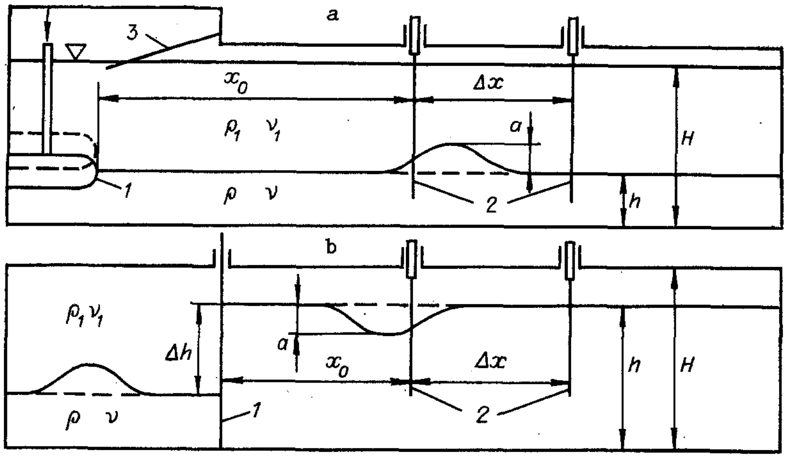

Fig. 1

As mentioned, a theoretical analysis is carried out in [1] for covered immiscible fluids, and the following equation is obtained for K:

$$K_1 = A_1\left[\left(1+\frac{2h}{B}\right)\frac{H-h}{h}+\beta_1\frac{H^2}{h(H-h)}+\gamma_1\frac{2h}{B}\right], \tag{2}$$

in which the three terms in the brackets describe the losses in the lower fluid (at the bottom and walls), at the interface (above and below), and in the upper fluid (at the walls); B is the width of the channel; the parameters A_1, β_1, and γ_1 are defined in [1]. Friction at the underside of the interface is assumed to be equal to the friction at the bottom, and the friction at the topside is assumed to be equal to the friction at the coverplate.

Even though friction presumably exists at the coverplate but is omitted in the last term of Eq. (2), it must be borne in mind that the parameters A_1, β_1, and γ_1 are derived with allowance for this friction. It is therefore not quite correct to compare the calculations and the experiments in [1], because this model was tested experimentally in a free-surface channel.

The analysis in [3] is carried out for immiscible fluids with a free surface. Interfacial friction is taken into account more rigorously here, and the following expression is obtained for K:

$$K_2 = A_2\left[\alpha_2\left(1+\frac{2h}{B}\right)\frac{H-h}{h}+\beta_2\frac{H^2}{h(H-h)}+\gamma_2\frac{2h}{B}\right]. \tag{3}$$

The terms in the brackets have the same significance as in Eq. (2) (A_2, α_2, β_2, and γ_2 are defined in [3]). The experimental data with which the results of [3] are compared were obtained in experiments with miscible fluids [5], when interfacial friction did not exist. This apparently explains the slower wave attenuation in the experiments than in the calculations.

In the present study we used immisible fluids and carried out the experiments both with a free surface and under a coverplate. We therefore augment Eqs. (2) and (3) with the equations

$$K_3 = A_1\left[\left(1+\frac{2h}{B}\right)\frac{H-h}{h}+\beta_1\frac{H^2}{h(H-h)}+\gamma_1\left(1+\frac{2(H-h)}{B}\right)\frac{h}{H-h}\right]; \tag{4}$$

$$K_4 = A_2\left[\alpha_2\left(1+\frac{2h}{B}\right)\frac{H-h}{h}+\beta_2\frac{H^2}{h(H-h)}+\gamma_2\left(1+\frac{2(H-h)}{B}\right)\frac{h}{H-h}\right], \tag{5}$$

in which friction at the coverplate is added to the last term. Equations (2) and (3) must be used in the free-surface calculations.

It will be shown below that the model of [1] does not yield satisfactory agreement with the experimental data, but calculations according to the model of [3] are in sufficiently good agreement with them. Computer calculations according to the model of [4] by Khabakhpashev using data from one of the experiments gave results close to those in [3].

The theoretical calculations are compared in Figs. 2-5 with experimental data on the amplitude attenuation of solitary internal waves and the evolution of their waveform. It should be noted that some of the experimental data refer to waves reflected from the end

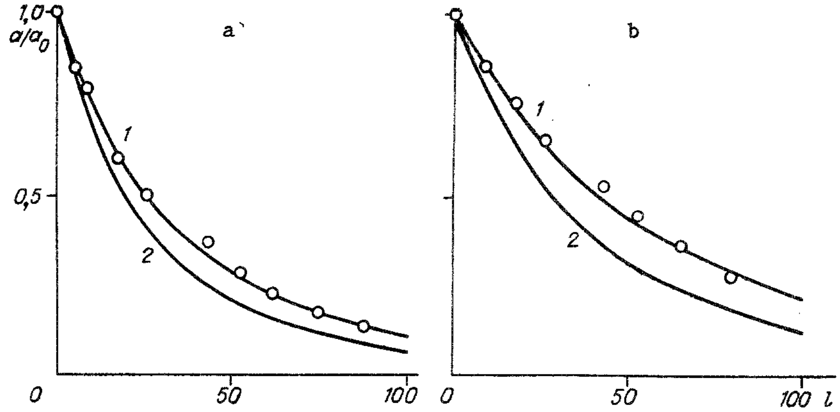

Fig. 2

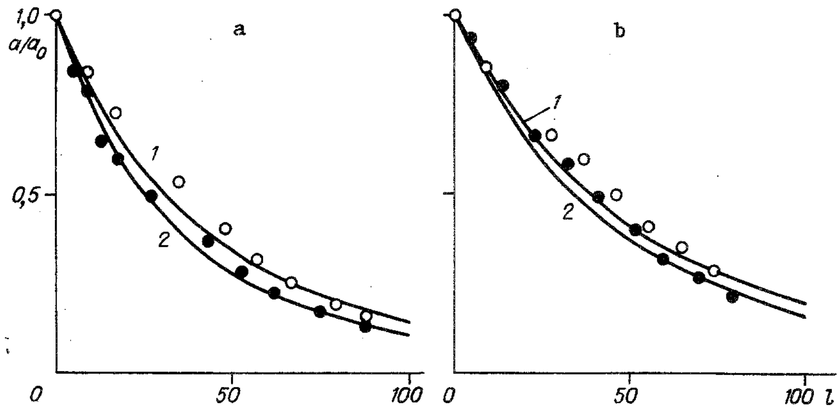

Fig. 3

walls of the experimental apparatus. The wave distance traversed $\ell = (x - x_0)/H$ is plotted along the horizontal axis in Figs. 2-4, and the wave amplitude relative to the amplitude at the points x_0 is plotted along the vertical axis. In all the figures (except Fig. 3b) $\rho_1 = 0.8$ g/cm³, $\nu_1 = 0.0162$ cm²/sec and $\rho = 1$ g/cm³, $\nu = 0.0108$ cm²/sec.

Figure 2 shows data on the attenuation of solitary waves in the form of humps (a) and troughs (b) in the free-surface experiments; curves 1 and 2 are calculated according to Eqs. (1), (3) and Eqs. (1), (2), respectively, and the points represent the experimental data; H = 5.7 cm, B = 6 cm, and x_0 = 160 cm. In Fig. 2a, h = 1.45 cm and a_0 = 0.51 cm; in Fig. 2b h = 4.25 cm and a_0 = −0.615 cm. We see that the attenuation of a trough wave is slower, because the contribution from friction at the channel bottom is smaller.

Figure 3 illustrates how the attenuation of a hump wave varies as a function of the channel width (a) and how the attenuation of a trough wave varies for different densities of the upper fluid (b); the curves are calculated according to Eqs. (1) and (3), and the points represent the experimental data, H = 5.7 cm. In Fig. 3a, curve 1 and the light points were obtained for h = 1.45 cm, B = 17.5 cm, x_0 = 250 cm, and a_0 = 0.53 cm; curve 2 and the dark points were obtained for respective values of 1.45, 6, 160, and 0.51 cm. In Fig. 3b, curve 1 and the light points were obtained for B = 6 cm, h = 4.25 cm, x_0 = 260 cm, a_0 = −0.51 cm, ρ_1 = 0.8 g/cm³, and ν_1 = 0.0162 cm²/sec; curve 2 and the dark points were obtained for the respective values of 6, 4.25, 210, −0.64 cm; 0.9 g/cm³; 0.0141 cm²/sec.

Figure 4 shows the attenuation of solitary internal waves in the form of troughs in experiments with a coverplate; curves 1 and 2 were calculated according to Eqs. (1), (5) and Eqs. (1), (4); the points represent the experimental data, H = 6 cm, h = 4.5 cm. In Fig. 4a, x_0 = 210 cm, a_0 = −0.59 cm, and B = 6 cm; in Fig. 4b, the corresponding values are 48, −0.88, and 20 cm. It must be noted that the parameters A_2, α_2, β_2, and γ_2 in the model of [3] were obtained in the free-surface case, and so it is not quite correct to use Eq. (5). Nonetheless, fairly good agreement is observed between the calculated and experimental data.

Figure 5 illustrates the evolution of the profile of the same solitary wave in the form of a trough at distances of 48 (a), 192 (b), 352 (c), and 504 cm (d) from the wave generator in the covered experiments; all the other parameters are the same as in Fig. 4b. Here the

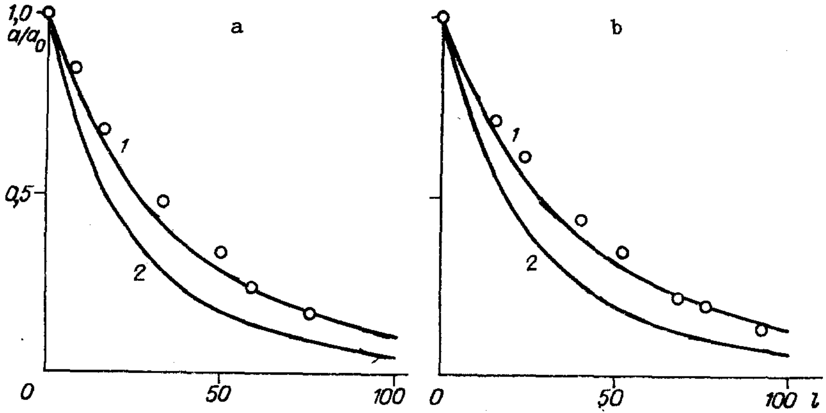

Fig. 4

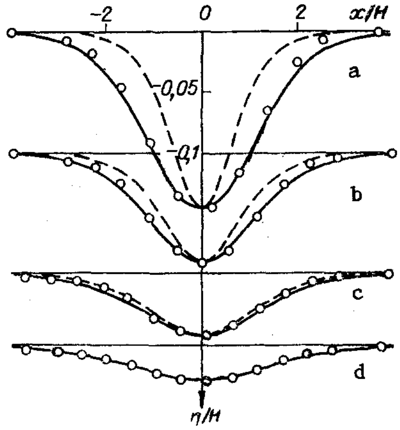

Fig. 5

horizontal coordinate x is plotted along the horizontal axis, and the deviation η of the interface from the equilibrium position is plotted along the vertical axis, both normalized to the depth of the fluids H; the solid curves represent the wave profile calculated according to the model of [7], the dashed curves represent the same according to [1], and the points represent the experimental data. The experimental values of the wave amplitude were used in the theoretical calculations. It is seen that the model of [1] provides a good description only for a small-amplitude wave, whereas the model of [7] is valid for any waves. The calculations according to the model of [5] give results close to those of [1].

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